Gravitational wave signal from Massive gravity

[arXiv:1208.5975]

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Introduction

- Probe massive gravity theories by gravitational wave observations

- We assume EoM of GW is modified by time-dependent graviton mass:

$$\ddot{\gamma}_k + 3H\dot{\gamma}_k + \left(\frac{k^2}{a(t)^2} \boxed{+ M_{GW}^2(t)} \right) \gamma_k = 0$$

➔ Argue how to detect $M_{GW}(t)$ from observational signals

Introduction

- Massive gravity as IR modified gravity
  - Self-accelerating universe without dark energy
  - Various modifications to gravitation
- Stochastic gravitational wave background
  - Gravitational wave generated in inflationary era
  - Target for upcoming GW observations
  - A probe for massive gravity theories

Contents

Massive gravity theories

- Fierz-Pauli massive gravity (1939)

$$S = \frac{M_{Pl}^2}{2} \int d^4x \left[R - \frac{1}{4} m^2 \left(h_{\mu\nu} h^{\mu\nu} - h^2 \right) \right]$$

$$\left(h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu} \right)$$

  - Works well at linear order
  - Suffers from Ghost instability at non-linear order

  → Many improved theories have been proposed

Massive gravity theories

- Massless gravity (GR): 2 tensor modes
- Massive gravity: 1 scalar + 2 vector + 2 tensor modes
  - Modifications by scalar modes
    → Probed by solar system tests etc.
  - Modifications to tensor modes
    → Affects gravitational wave propagation

➔ Gravitational wave observations will be relevant if

✓ Scalar and vector modes behaves exactly same as GR

✓ Tensor modes are modified by the graviton mass

Massive gravity theories

- Examples:
  - Non-linear extension of Fierz-Pauli massive gravity

(de Rham, Gabadadze & Tolley 2011)

$$S = M_{Pl}^2 \int d^4x \sqrt{-g} \left[\frac{R}{2} + m_g^2 \left(\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4 \right) \right]$$

$$\mathcal{L}_2 = \frac{1}{2} \left([\mathcal{K}]^2 - [\mathcal{K}^2] \right), \quad \mathcal{L}_3 = \frac{1}{6} \left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3] \right),$$

$$\mathcal{L}_4 = \frac{1}{24} \left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4] \right),$$

$$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - \left(\sqrt{g^{-1} \hat{f}} \right)^\mu{}_\nu, \quad [\mathcal{K}] = \mathrm{tr}\mathcal{K}, \quad \hat{f}_{\mu\nu} = f_{ab} \partial_\mu \phi^a \partial_\nu \phi^b$$

Massive gravity theories

- Examples:
  - Non-linear extension of Fierz-Pauli massive gravity

(de Rham, Gabadadze & Tolley 2011)

$$S = M_{Pl}^2 \int d^4x \sqrt{-g} \left[\frac{R}{2} + m_g^2 \left(\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4 \right) \right]$$

- No BD ghost even at non-linear level (Hassan & Rosen 2011)

- Effective Λ appears from mass terms
  → Self-accelerating FRW universe
- Cosmological perturbations

Massive gravity theories

- Examples:
  - Non-linear extension of Fierz-Pauli massive gravity
    (de Rham, Gabadadze & Tolley 2011)
    - Cosmological perturbations (Gümrükçüoğlu, Lin & Mukohyama 2011)
      - Scalar & vector perturbations:
        May behave exactly same as GR
      - Tensor perturbations:
        EoM in GR + Time-dependent mass term

$$\ddot{\gamma}_k + 3H\dot{\gamma}_k + \left(\frac{k^2}{a(t)^2} + M_{GW}^2(t)\right)\gamma_k = 0$$

Massive gravity theories

- Examples:
  - Lorentz-violating massive gravity theories

(Dubovsky 2004 etc)

$$S = \int dx^4 \sqrt{-g}\, [L_{\rm GR} + L_{\rm mass}]$$

$$L_{\rm mass} \simeq \frac{1}{4}\left\{m_0^2 h_{tt}^2 + 2m_1^2 h_{ti}^2 - m_2^2 h_{ij}^2 + m_3^2 h_{ii}^2 - 2m_4^2 h_{tt} h_{ii}\right\}$$

with $m_1 = 0, \quad m_0^2 = -3\gamma m_4^2, \quad \gamma\left(m_2^2 - 3m_3^2\right) = m_4^2 - \frac{1}{2}m_1^2$

➢ Cosmological perturbations

- Scalar perturbations: Rather Mild modification
- Vector perturbations: Behaves exactly same as GR
- Tensor perturbations: EoM in GR + Mass term

Massive gravity theories

- Examples:
  - Lo
    - 04 etc)
    - $i\}$
    - R
    - Tensor perturbations: EoM in GR + Mass term

Caveats:

- Instability of FRW solutions
  [De Felice+, . . .]
- Superluminarity
  [Deser & Waldron, . . .]
- Other constraints
  [Burrage+, . . .]

Massive gravity theories

- General model for which GW observation is relevant:
  - ✓Scalar and vector modes behaves exactly same as GR
  - ✓Tensor modes obey a ghost-free general action

$$I = \frac{M_{Pl}^2}{8} \int dt dx^3 N a^3 \sqrt{\Omega} \left[\frac{1}{N^2} \dot{\gamma}^{ij} \dot{\gamma}_{ij} + \gamma^{ij} \left(\sum_{n=0}^{\infty} c_n(t) \frac{\triangle^n}{a^{2n}} \right) \gamma_{ij} \right]$$

$$\simeq \frac{M_{Pl}^2}{8} \int dt dx^3 N a^3 \sqrt{\Omega} \left[\frac{1}{N^2} \dot{\gamma}^{ij} \dot{\gamma}_{ij} + \frac{c_g^2(t)}{a^2} \gamma^{ij} \left(\triangle - 2K \right) \gamma_{ij} - M_{GW}^2(t) \gamma^{ij} \gamma_{ij} \right]$$

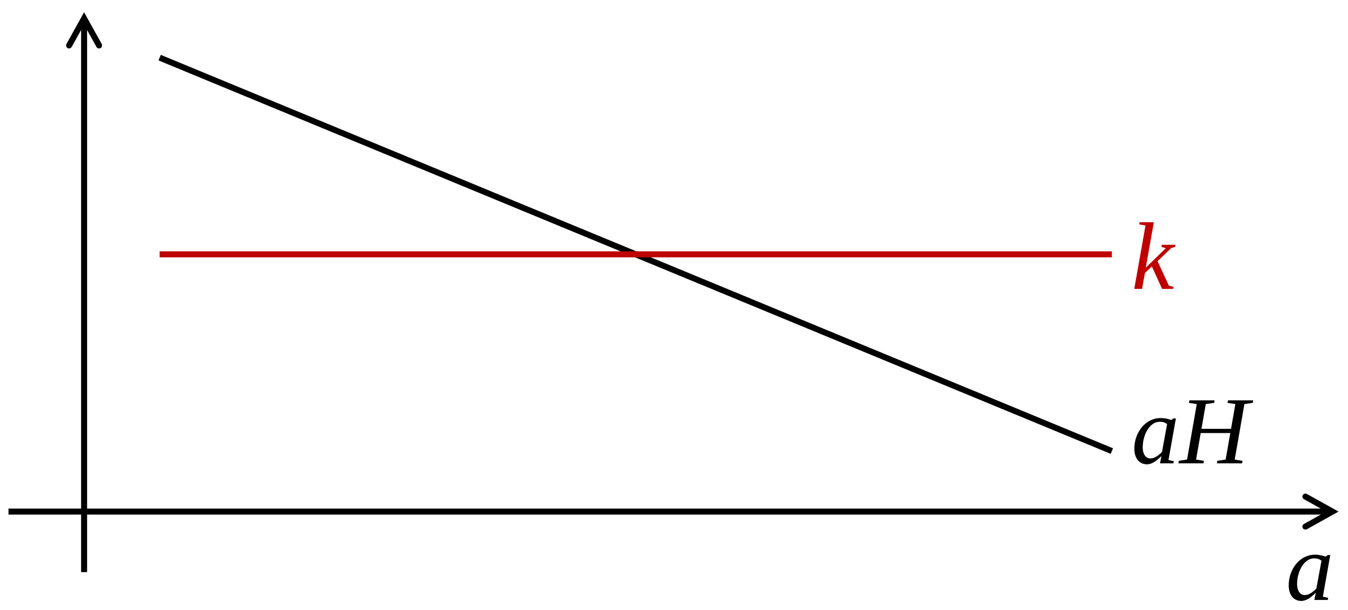
$$\Rightarrow \quad \bar{\gamma}_k'' + \left(c_g^2(t) \left(k^2 + 2K \right) - \frac{a''}{a} + a^2 M_{GW}^2(t) \right) \bar{\gamma}_k = 0$$

✓ Probe $M_{GW}(t)$ by observations of stochastic gravitational wave

Evolution of gravitational wave

- Pure GR

$$\ddot{\gamma}_k + 3H^2\dot{\gamma}_k + \left(\frac{k^2}{a(t)^2} + M_{GW}^2(t)\right)\gamma_k = 0$$



Evolution of gravitational wave

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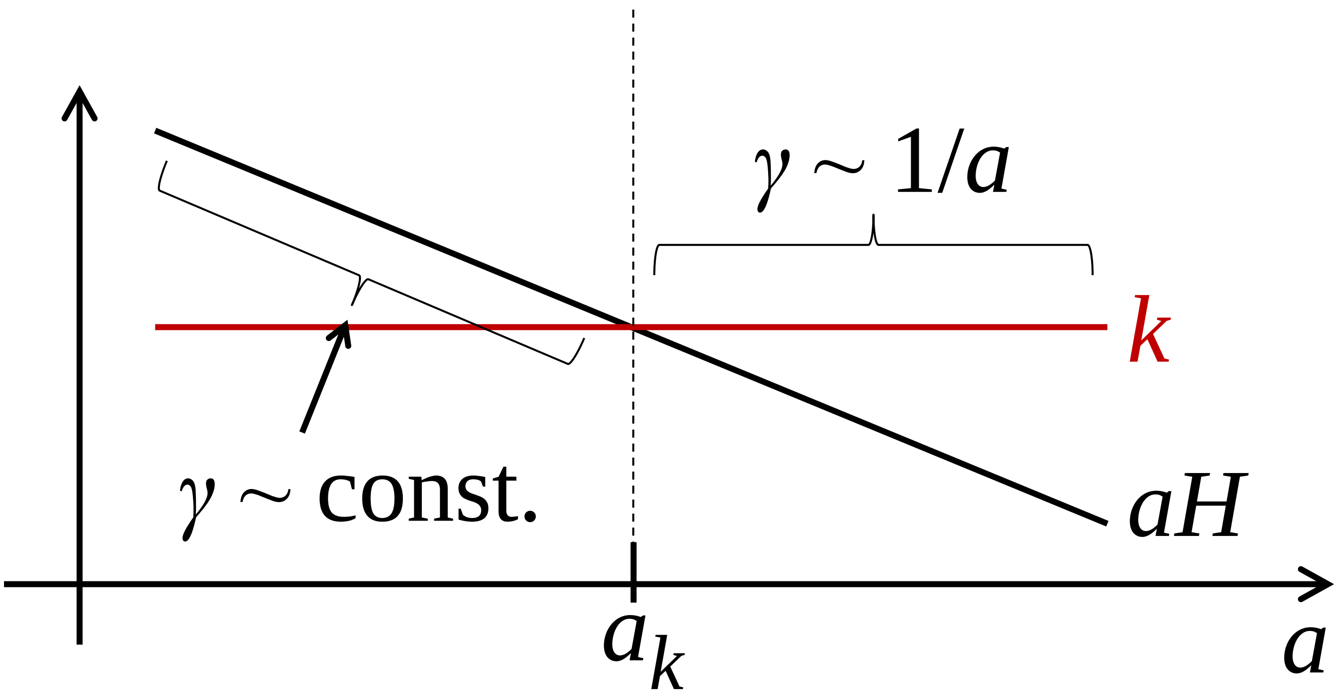


Evolution of gravitational wave

- Pure GR

$$\ddot{\gamma}_k + 3H^2\dot{\gamma}_k + \left(\frac{k^2}{a(t)^2} + M_{GW}^2(t)\right)\gamma_k = 0$$

  - WKB solution

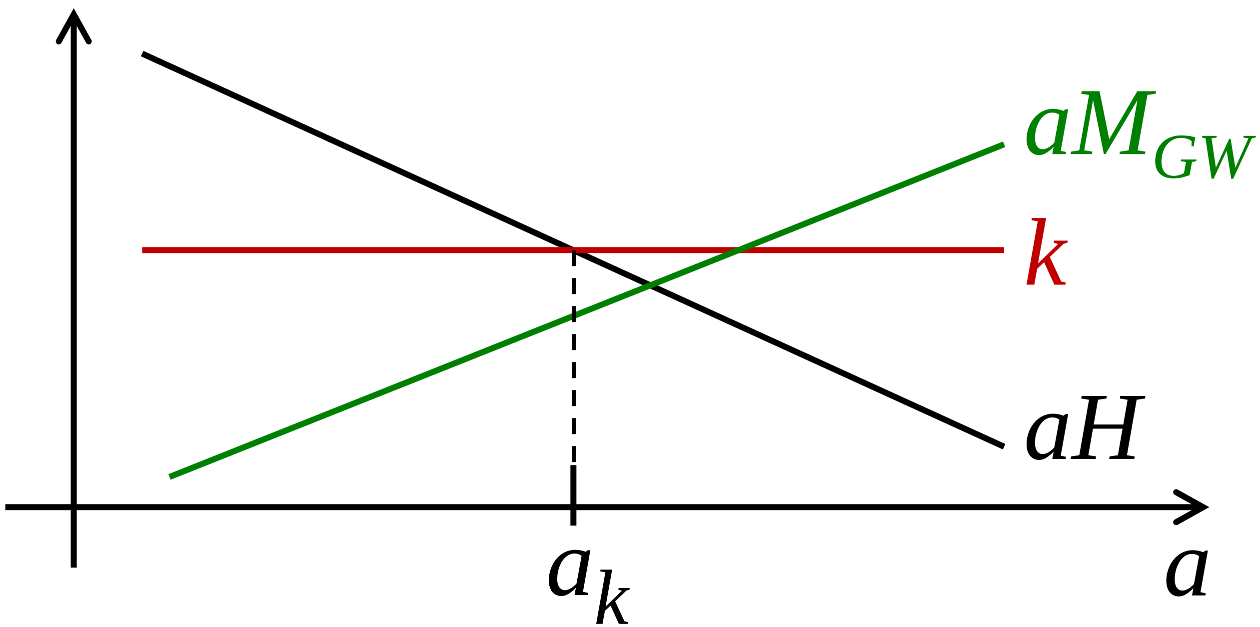
$$\gamma_k = A(k)\frac{a_k}{a(t)}\exp\left(i\int\frac{k}{a}dt\right)$$

$$\left[A(k) \equiv \frac{H_*}{M_{Pl}k^{3/2}} \ : \ \text{Primordial amplitude}\right]$$

Evolution of gravitational wave

- Pure GR + Graviton mass term

$$\ddot{\gamma}_k + 3H^2\dot{\gamma}_k + \left(\frac{k^2}{a(t)^2} + M_{GW}^2(t)\right)\gamma_k = 0$$

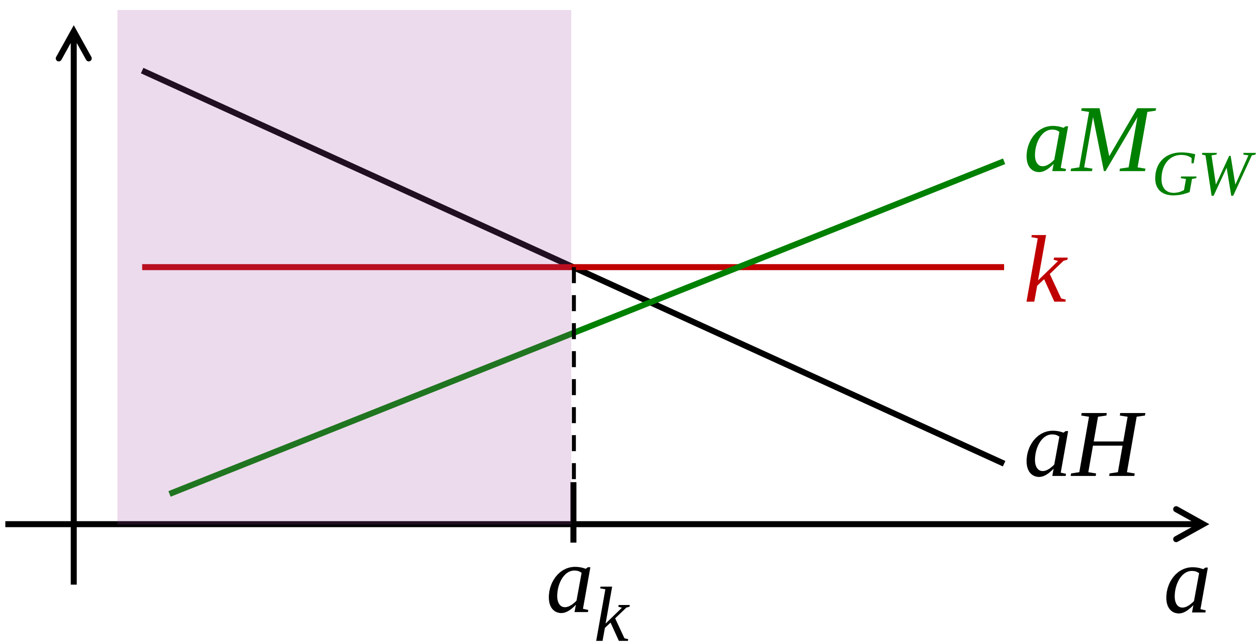


Evolution of gravitational wave

- Early time:

$$\ddot{\gamma}_k + 3H^2\dot{\gamma}_k + \left(\frac{k^2}{a(t)^2} + M_{GW}^2(t)\right)\gamma_k = 0$$

→ $\gamma_k \approx$ constant

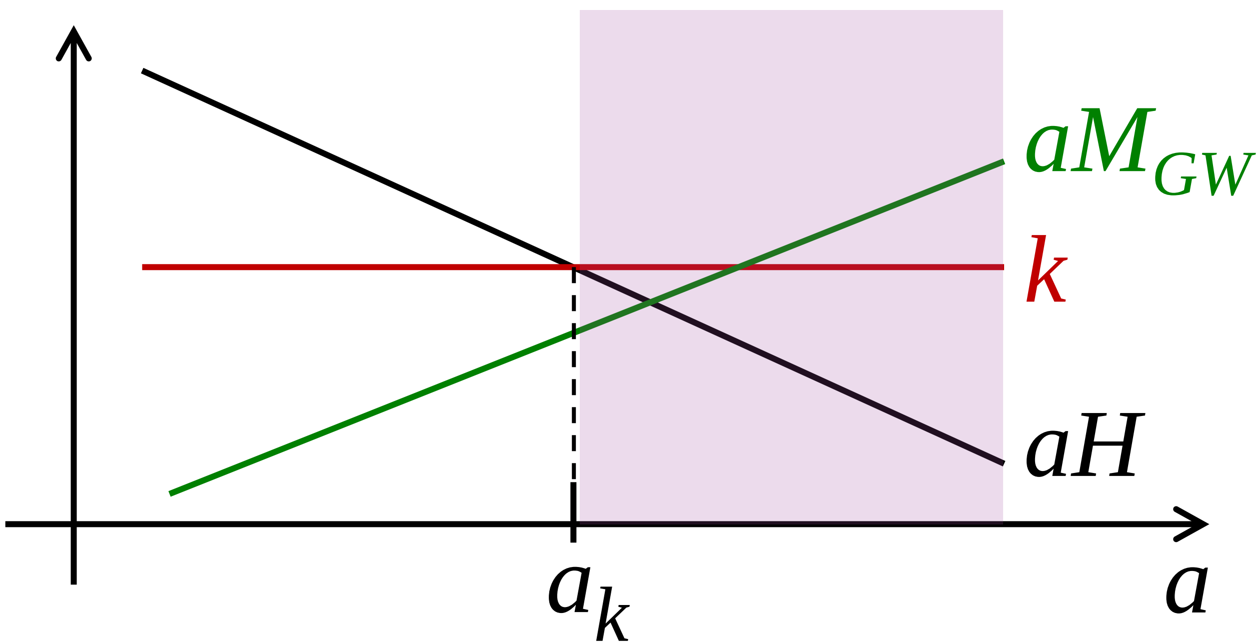


Evolution of gravitational wave

- Late time:

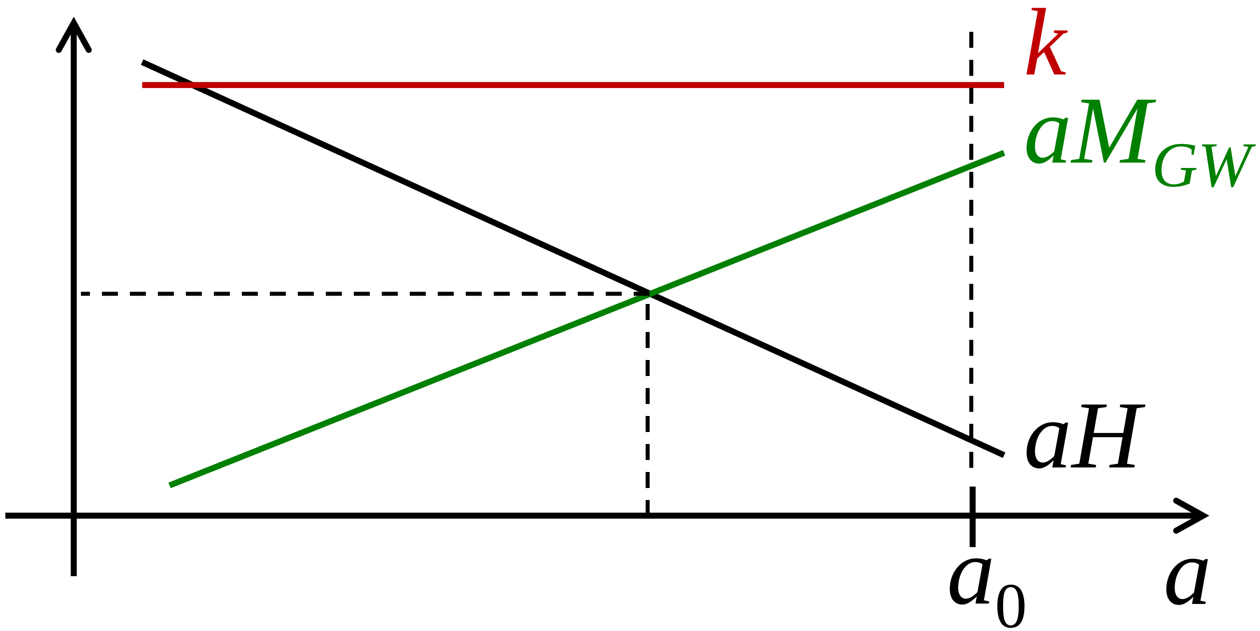
$$\ddot{\gamma}_k + 3H^2\dot{\gamma}_k + \left(\frac{k^2}{a(t)^2} + M_{GW}^2(t)\right)\gamma_k = 0$$

→ γ_k oscillates with $\omega(t) = \sqrt{\frac{k^2}{a^2} + M_{GW}^2(t)}$



Evolution of gravitational wave

- Pure GR + Mass term:
  - Large k : Same as pure GR
  - Medium k : Suppression of γ near today
  - Small k : Dominated by $M_{GW}(t)$



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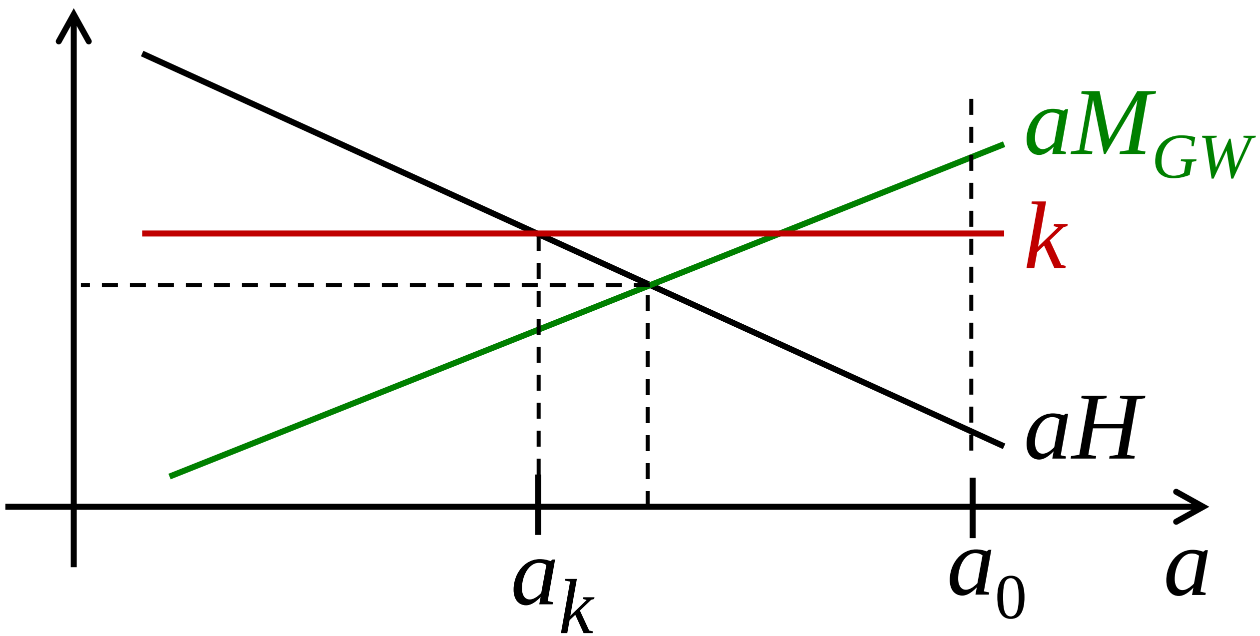

Evolution of gravitational wave

- Pure GR + Mass term:

$$\ddot{\gamma}_k + 3H\dot{\gamma}_k + \boxed{\left(\frac{k^2}{a(t)^2} + M_{GW}^2(t)\right)}\gamma_k = 0$$

$$\equiv \omega^2(t)$$

  - WKB solution

$$\gamma_k = A(k)\sqrt{\frac{a_k^3\ \omega_k}{a(t)^3\omega(t)}}\exp\left(i\int \omega(t)dt\right)$$

$$\left[\text{GR:}\quad \gamma_k = A(k)\frac{a_k}{a(t)}\exp\left(i\int \frac{k}{a}dt\right)\right]$$

Evolution of gravitational wave

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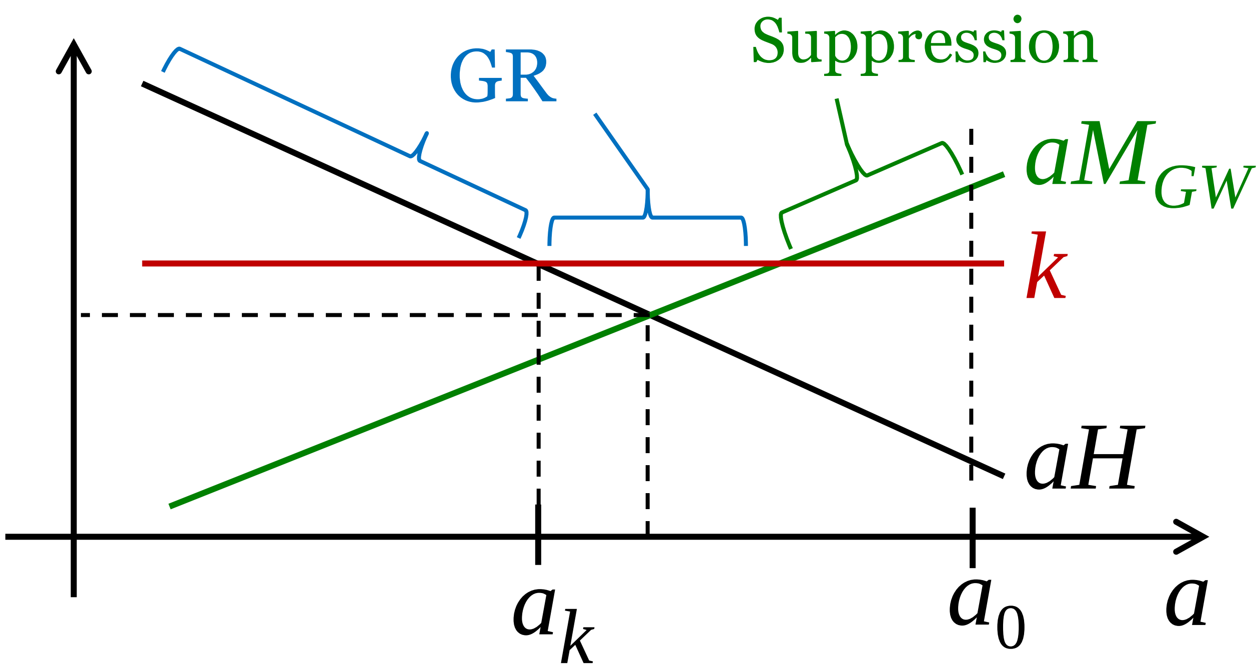

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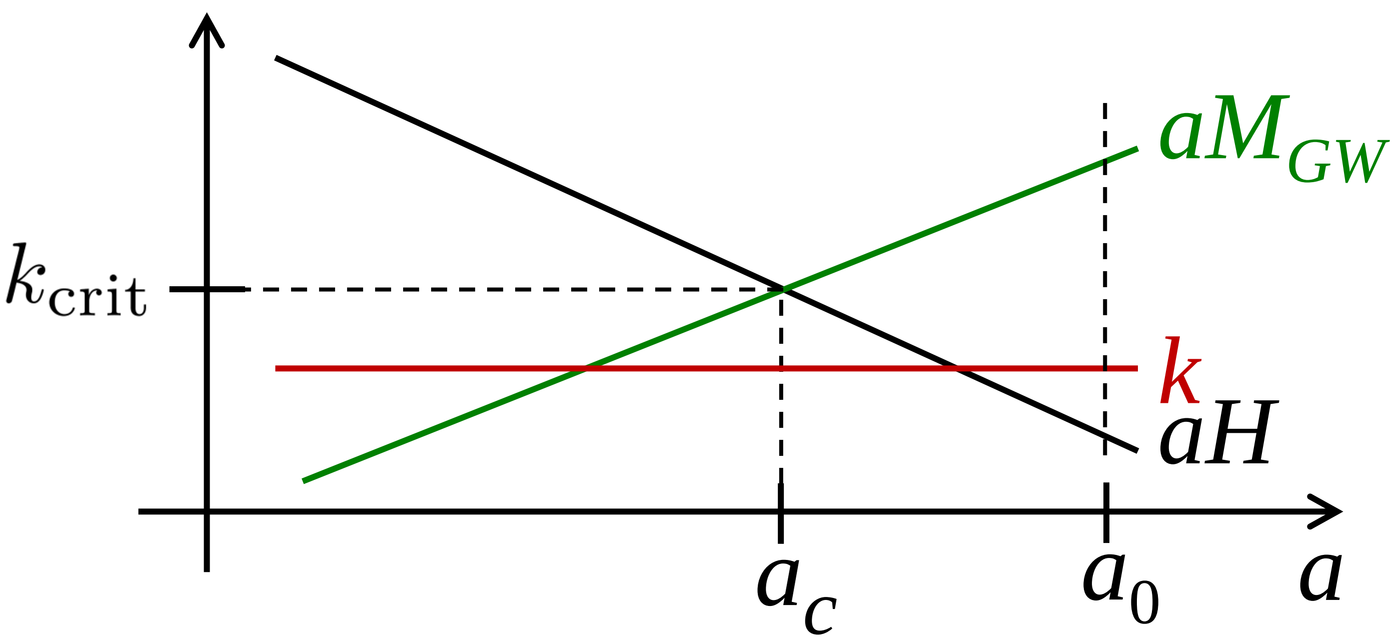


Evolution of gravitational wave

- Pure GR + Mass term:

$$\ddot{\gamma}_k + 3H\dot{\gamma}_k + \underbrace{\left(\frac{k^2}{a(t)^2} + M_{GW}^2(t) \right)}_{\equiv \omega^2(t)} \gamma_k = 0$$

  - WKB solution

$$\gamma_k = A(k) \sqrt{\frac{a_k^3 \ \omega_k}{a(t)^3 \, \omega(t)}} \exp\left(i \int \omega(t) dt \right)$$

$$\Rightarrow \quad |\gamma_k(t_0)| = A(k) \sqrt{\frac{a_c^3 M_{GW}(t_c)}{a_0^3 M_{GW}(t_0)}}$$

$$\left[A(k) \equiv \frac{H_*}{M_{Pl} k^{3/2}} \ : \ \text{Primordial amplitude} \right]$$

Observed spectrum

- We've discussed **power spectrum w.r.t. *k***:

$$\mathcal{P}(k) \equiv \frac{d}{d\ln k}\langle \gamma_{ij}\gamma^{ij}\rangle\bigg|_{t=t_0} = \frac{2k^3}{\pi^2}\left|\gamma_k(t_0)\right|^2$$

- Observatories measures **power spectrum w.r.t. *ω***:

$$\mathcal{P}(\omega_0) \equiv \frac{d}{d\ln \omega_0}\langle \gamma_{ij}\gamma^{ij}\rangle\bigg|_{t=t_0} = \frac{d\ln k}{d\ln \omega_0}\mathcal{P}(k(\omega))$$

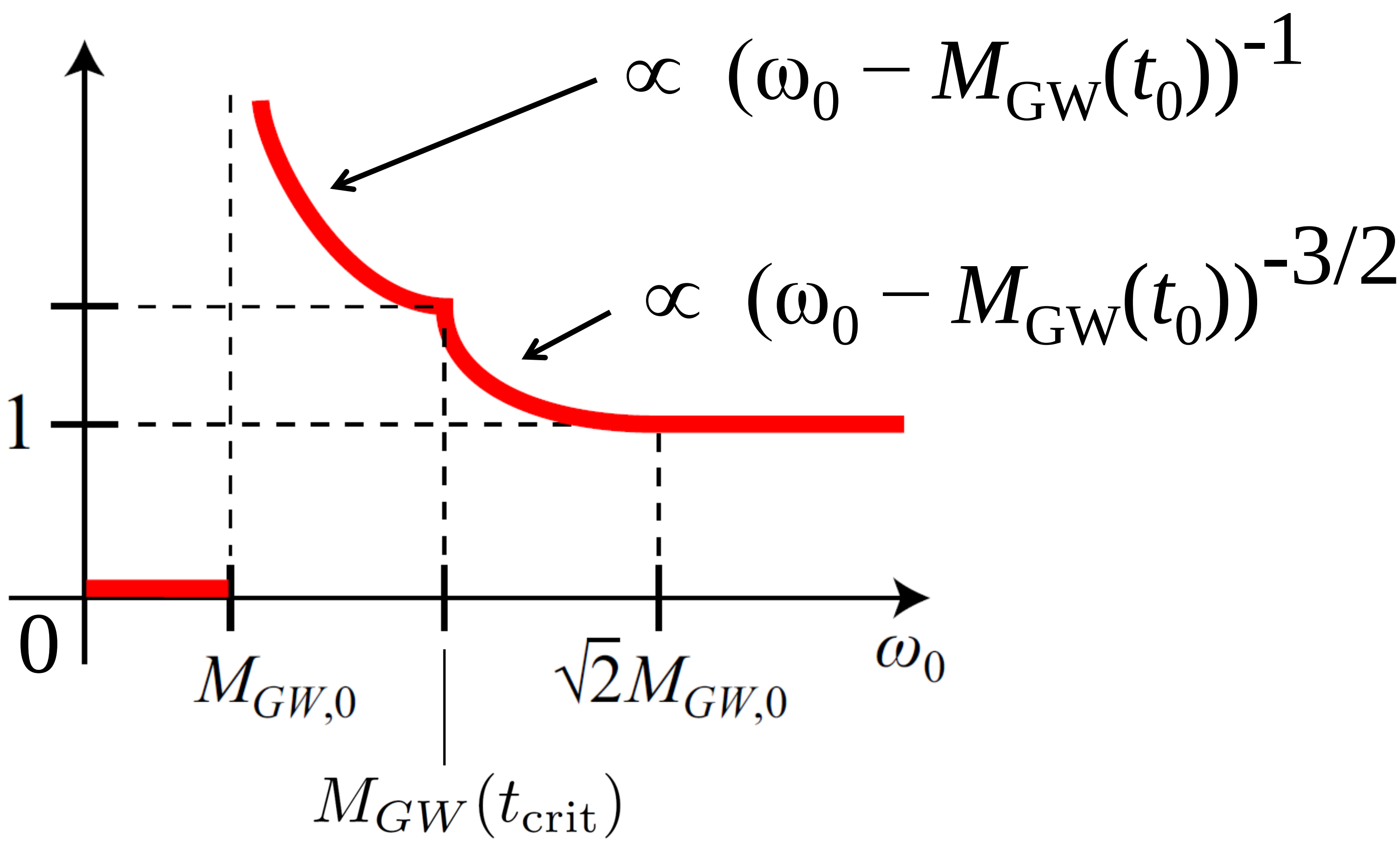
$$\left(\omega_0^2 = \frac{k^2}{a_0^2} + M_{GW}^2(t_0)\right)$$

$$\uparrow$$

$$\frac{\omega_0^2}{\omega_0^2 - M_{GW}^2(t_0)}$$

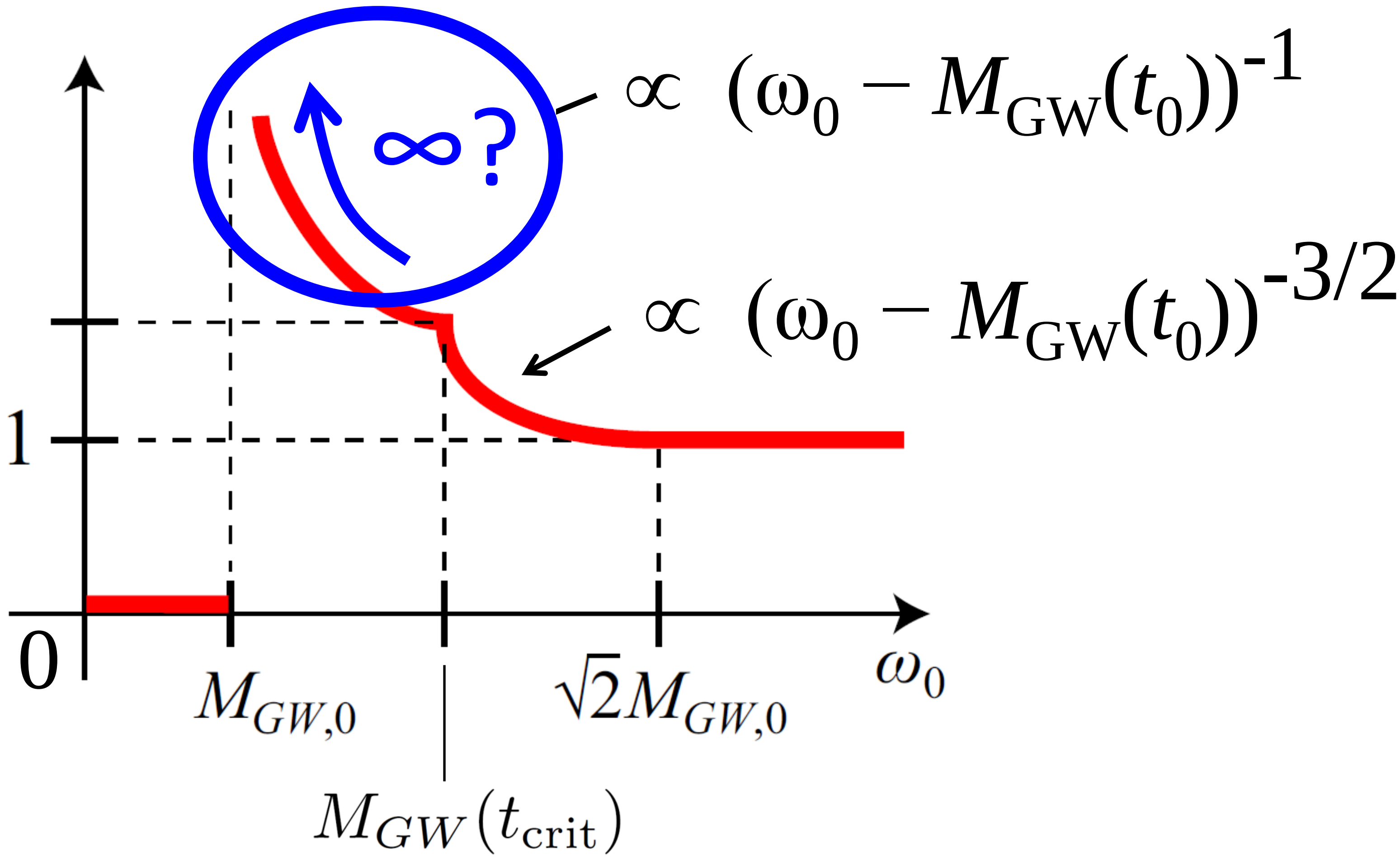
Observed spectrum

- ($\mathcal{P}(\omega)$ in MG) / ($\mathcal{P}(\omega)$ in GR) for the same ω



Observed spectrum

- ($\mathcal{P}(\omega)$ in MG) / ($\mathcal{P}(\omega)$ in GR) for the same ω



Observed spectrum

- Peak in $\mathcal{P}(\omega)$ is sensitive to $\mathcal{P}_{\rm prim}(k)$:

$\propto \mathcal{P}_{\rm prim}(k)$ for $\omega \to M_{GW}$ & $k \to 0$

$$\mathcal{P}(\omega_0) = \boxed{\frac{d\ln k}{d\ln\omega_0}}\,\boxed{\mathcal{P}\big(k(\omega_0)\big)} \propto k^{-2}\mathcal{P}_{\rm prim}(k)\big|_{k=k(\omega_0)}$$

$$\left[\ \omega_0^2 = \frac{k^2}{a_0^2} + M_{GW}^2(t_0) \quad \Rightarrow \quad \frac{d\ln k}{d\ln\omega_0} = \left(\frac{a_0\omega_0}{k}\right)^2\ \right]$$

➤ If $\mathcal{P}_{\rm prim}(k)$ is flat for $k \to 0$, $\mathcal{P}(\omega)$ diverges at $\omega = M_{\rm GW}(t_0)$

Observed spectrum

- Peak in $\mathcal{P}(\omega)$ is sensitive to $\mathcal{P}_{\rm prim}(k)$:

$\propto \mathcal{P}_{\rm prim}(k)$ for $\omega \to M_{GW}$ & $k \to 0$

$$\mathcal{P}(\omega_0) = \boxed{\frac{d\ln k}{d\ln\omega_0}}\boxed{\mathcal{P}\left(k(\omega_0)\right)} \propto k^{-2}\mathcal{P}_{\rm prim}(k)\Big|_{k=k(\omega_0)}$$

$$\left[\ \omega_0^2 = \frac{k^2}{a_0^2} + M_{GW}^2(t_0) \ \Rightarrow\ \frac{d\ln k}{d\ln\omega_0} = \left(\frac{a_0\omega_0}{k}\right)^2\ \right]$$

➢ If $\mathcal{P}_{\rm prim}(k)$ has a **cutoff near k = 0**,

$$\text{Peak height} \ \sim \lim_{k\to k_{\rm cutoff}} k^{-2}\mathcal{P}_{\rm prim}(k) < +\infty$$

ex.)
- $N_{\text{e-fold}} \approx 65$ → $k_{\rm cutoff}$ = 1 H_0
- $M_{\rm GW}(t_0)$ =10^{-8} Hz ≈ 10^9 H_0

→ ($\mathcal{P}(\omega)$ in MG) / ($\mathcal{P}(\omega)$ in GR) ~10^{23} at the peak

Observed spectrum

- Peak in $\mathcal{P}(\omega)$ is sensitive to $\mathcal{P}_{\text{prim}}(k)$:

$\propto \mathcal{P}_{\text{prim}}(k)$ for $\omega \to M_{GW}$ & $k \to 0$

$$\mathcal{P}(\omega_0) = \boxed{\frac{d\ln k}{d\ln\omega_0}}\,\boxed{\mathcal{P}\left(k(\omega_0)\right)} \propto k^{-2}\mathcal{P}_{\text{prim}}(k)\Big|_{k=k(\omega_0)}$$

$$\left[\ \omega_0^2 = \frac{k^2}{a_0^2} + M_{GW}^2(t_0) \quad \Rightarrow \quad \frac{d\ln k}{d\ln\omega_0} = \left(\frac{a_0\omega_0}{k}\right)^2\ \right]$$

- Peak height → $\lim_{k\to+0} k^{-2}\mathcal{P}_{\text{prim}}(k)$
  → small k cutoff of $\mathcal{P}_{\text{prim}}(k)$
- Peak location → $M_{GW}(t_0)$
- Peak shape → $M_{GW}(t_{\text{crit}})$

Observed spectrum

- Sensitivity range:
  - LISA: $10^{-4} \sim 1$ Hz
  - DECIGO: $10^{-1} \sim 1$ Hz
  - SKA, PPTA: $10^{-8} \sim$ Hz
- Current bound:
  - $M_{GW}(t_0) < 10^{-5}$ Hz from binary pulsar timing

[Finn & Sutton 2002]

➔ GW signal will be observable if

$$10^{-8}\ \text{Hz} < M_{GW}(t_0) < 10^{-5}\ \text{Hz}$$

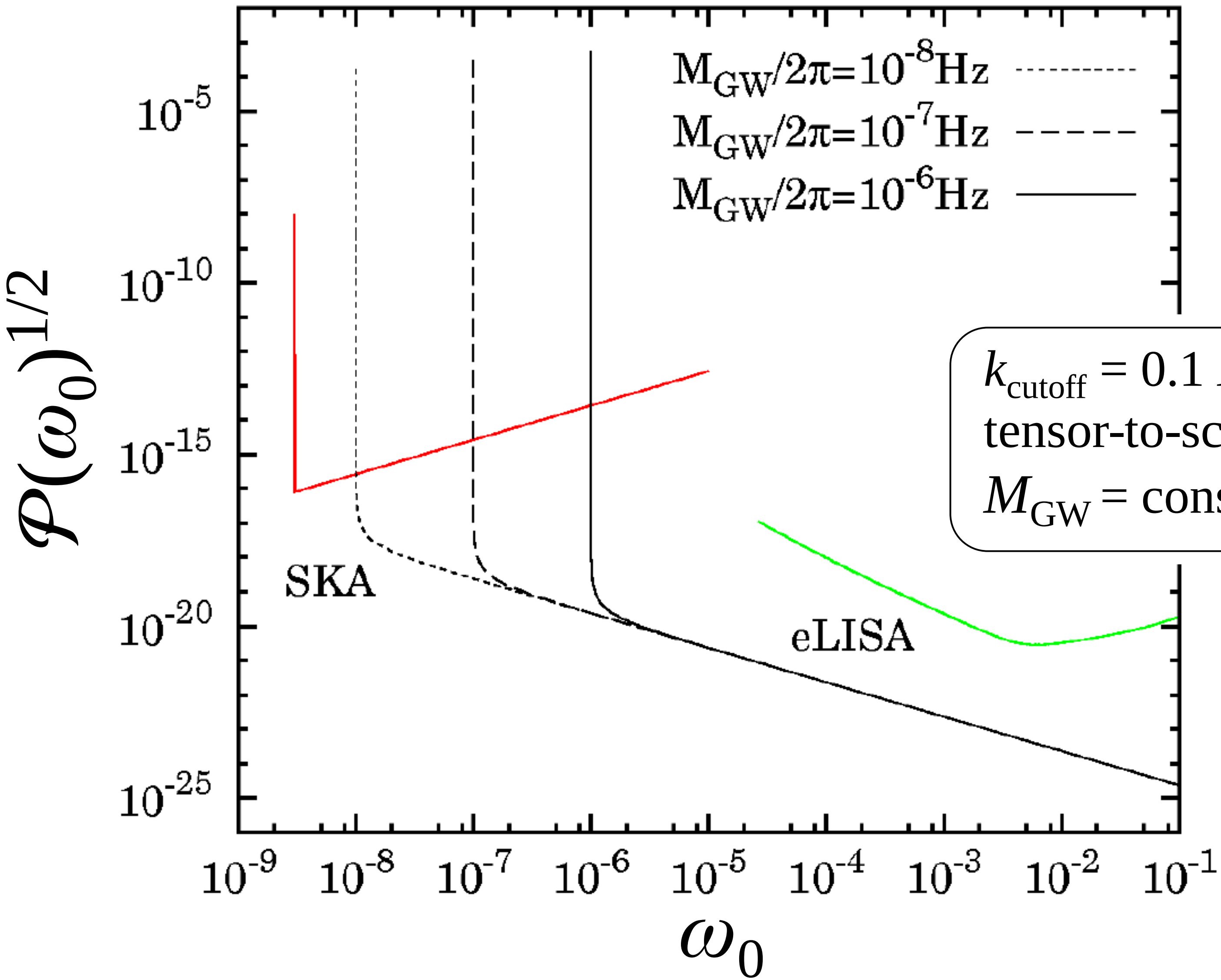
Observed spectrum

- Amplification from GR:

$$\frac{\mathcal{P}^{\mathrm{MG}}(\omega_0)}{\mathcal{P}^{\mathrm{GR}}(\omega_0)} \sim \frac{a_c^2 k_c}{a_{k_0}^{GR^2} k_0} \left(\frac{\omega_{\mathrm{cutoff}}^2}{M_{GW,0}^2} - 1 \right)^{-1}$$

• $M_{\mathrm{GW}}(t_0)$ = **10^{-8} Hz** $\approx 10^9\,\mathrm{H}_0$ • $k_{\mathrm{cutoff}} = 1\,\mathrm{H}_0$	$\sim 10^{23}$
• $M_{\mathrm{GW}}(t_0)$ = **10^{-4} Hz** $\approx 10^{13}\,\mathrm{H}_0$ • $k_{\mathrm{cutoff}} = 1\,\mathrm{H}_0$	$\sim 10^{35}$

Observed spectrum



$k_{cutoff} = 0.1\ H_0$
tensor-to-scalar ratio = 0.1
M_{GW} = constant

Summary

- Probe time-dependent mass of general massive gravity theories by gravitational wave observations

- GW direct observations:
  - Sharp peak in $\mathcal{P}(\omega)$
  - $M_{GW}(t)$ at $t = t_0$ & t_{crit}

  - Peak height → $\lim_{k \to +0} k^{-2} \mathcal{P}_{\mathrm{prim}}(k)$
    → small k cutoff of $\mathcal{P}_{\mathrm{prim}}(k)$
  - Peak location → $M_{GW}(t_0)$
  - Peak shape → $M_{GW}(t_{\mathrm{crit}})$

- Other probes for $M_{GW}(t)$?
  - GW → CMB polarizations
    - Suppression at lower multipoles: [Dubovsky et al. 2009]

$$\ell < 10^{-3} \times M_{GW}(t_{\mathrm{rec}})/H_0$$

→ $M_{GW}(t)$ at recombination

  - $\Omega_{GW} h^2 \propto \omega^2 \mathcal{P}(\omega) \sim M_{GW}(t)^2 \mathcal{P}(\omega)$ → BBN constraint?